B.A./B.Sc. 6th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH6DSE41

(Bio Mathematics)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) What is the carrying capacity in logistic growth model?

(b) What is the Allee effect? Explain with an example.

(c) Discuss Gompertz growth model.

(d) Write a short note on Bacterial growth in a Chemostat.

(e) Determine when the steady state of the following equation is stable:

$$x_{n+1} = \frac{1}{2 + x_n}, n = 0, 1, 2, \ldots$$

(f) Find equilibrium solution of difference equation $x_{t+1} = rx_t(1 - x_t)$.

(g) Explain a continuous age-structured model.

(h) Discuss a simple discrete prey predator model.

(i) Explain the self-crowding effect in logistic growth model.

(j) What is intra-species competition? Explain with an example.

(k) Define a SIR model with generalized assumptions.

(l) What is the jury stability condition? Explain with an example.

(m) Write down the assumptions of density dependent growth models with harvesting.

(n) Discuss limit cycles with an example in the context of biological scenario.

(o) What are the assumptions of the Nicholson Bailey model?

2. Answer *any four* questions: 5×4=20

(a) Find the fixed points of the following model and carry out a linear stability analysis at (0, 0).

$$\frac{dN_1}{dt} = r_1 N_1 \left[1 - \frac{N_1}{K_1} - b_{12}\frac{N_2}{K_1}\right]$$
$$\frac{dN_2}{dt} = r_2 N_2 \left[1 - \frac{N_2}{K_2} - b_{21}\frac{N_1}{K_2}\right]$$

where r_1, K_1, r_2, K_2, b_{12} and b_{21} are all positive constants and have their usual meanings.

(b) Obtain the condition under which the following model will have a positive interior equilibrium point.

$$\frac{dx}{dt} = x(1 - x - py)$$
$$\frac{dy}{dt} = y(1 - y - qx), \text{where } p, q > 0$$

(c) Consider the following non-linear difference equation for population growth:

$$x_{n+1} = \frac{kx_n}{b + x_n}, b, k > 0$$

Does equation have a steady state? If so, is that steady state stable? 3+2

(d) Solve the following initial value problem by the method of characteristics:

$$u_t + vu_x = 0, t \in (0, \infty), x \in (-\infty, \infty), u(0, x) = \phi(x), x \in (-\infty, \infty)$$

(e) Discuss the phase plane analysis of a two dimensional system $\dot{x} = ax + by, \dot{y} = cx + dy$ when the eigenvalues are complex conjugate, where a, b, c and d are real.

(f) What is diffusion in mathematical model? Give an example of two species model with diffusion.

3. Answer *any two* questions: 10×2=20

(a) State and prove the Routh-Hurwitz criteria for a second order polynomial equation. Hence, discuss the nature of the roots of characteristic equation of the following differential equation: (2+4)+4

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + 3x = 0$$

(b) Consider the following system

$$\frac{dx}{dt} = x\left(1 - \frac{x}{k}\right) - dxy$$
$$\frac{dy}{dt} = exy - py$$

where k, d, e and p are all positive constants.

(i) Find corresponding steady states and Jacobean matrix around any fixed point.

(ii) Discuss the stability of interior steady state only. (3+2)+5

(c) Perform a local phase plane analysis of the following SIRS epidemic model:

$$\frac{dS}{dt} = -\frac{\beta}{N}SI - v(N - S - I),$$
$$\frac{dI}{dt} = \frac{\beta}{N}SI - \gamma I$$

where the parameters have their usual meanings. Find the equilibrium points and determine conditions for their local asymptotic stability. Consider two cases, $R_0 > 1$ and $R_0 \leq 1$, R_0 is the basic reproduction numbers. 4+6

(d) (i) Reduce a two species diffusion model into a linearized system around any specially uniform steady state.

(ii) Obtain the conditions for diffusive instability. 5+5
